

Electromagnetic radiation from vortex flow in type-II superconductors

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We show that a moving vortex lattice, as it comes to a crystal edge, radiates into a free space the harmonics of the washboard frequency, $\omega_0 = 2\pi v/a$, up to a superconducting gap, Δ/\hbar . Here v is the velocity of the vortex lattice and a is the intervortex spacing. We compute radiation power and show that this effect can be used for generation of terahertz radiation and for characterization of moving vortex lattices.

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Effects at the washboard frequency in the flux-flow regime of type II superconductors were first reported by Fiory [1]. Imposing an rf current at the harmonics of $\omega_0 = 2\pi v/a$ on top of the dc transport current, Fiory observed steps in the I-V characteristics of superconducting aluminum films. Here v is the vortex velocity and a is the vortex lattice parameter in the direction of \mathbf{v} . Similar observations were made in high-temperature superconductors [2, 3]. Larkin and Ovchinnikov [4], and Schmid and Hauger [5] have demonstrated that the effect originates from the coherent action of defects as they are passed periodically at the washboard frequency ω_0 by moving vortices. It is analogous to Shapiro steps in Josephson junctions in the presence of the ac current [6]. By analogy with the radiation from a Josephson junction [7, 8, 9], the inverse effect of the electromagnetic radiation from a moving vortex lattice at the washboard frequency should be expected. Radiation produced by a single vortex or vortex bundle crossing the edge of a superconductor was discussed by Dolgov and Schopohl [10]. They used the transition radiation approach to calculate the energy and spectrum of the broad-band electromagnetic pulse emitted by the bundle. In this Letter we show that the situation for a moving vortex lattice is completely different. As vortices come to the surface periodically with the frequency v/a , they radiate into free space the harmonics of ω_0 up to a frequency corresponding to the superconducting gap, Δ/\hbar . Unlike the rf component in the transport current produced by defects [1, 2, 3, 4, 5], the electromagnetic radiation from the surface does not disappear but becomes stronger when disorder weakens. It is generated by oscillating electric and magnetic fields of vortices near the surface and propagates into free space due to the continuity of tangential components of the fields at the surface. We study cases of ideal and disordered vortex lattice in small and large crystals as compared to the radiation wavelength. Radiation power is derived and shown to be within experimental reach. For a large crystal the effect is proportional to the radiating surface, while for a small crystal it is quadratic on the surface, that is, superradiant. The challenge for experiment is to satisfy the requirements of sufficient speed and appreciable correlation length of translational order. If these conditions are satisfied, the effect can be used for generation of electromagnetic radiation well into the terahertz range.

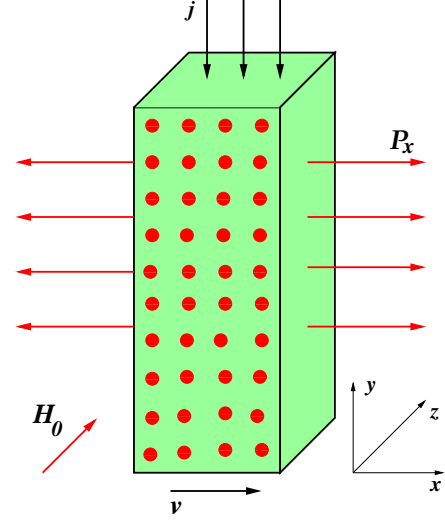


FIG. 1: (Color online) A moving vortex lattice radiates electromagnetic waves into a free space. Arrows indicate directions of the applied magnetic field \mathbf{H}_0 , transport current \mathbf{j} , vortex lattice velocity \mathbf{v} , and the Poynting vector \mathbf{P}_x . Vortices are shown by circles.

We study electromagnetic radiation from the boundaries, $x = 0, -L_x$, of a type-II isotropic superconductor, located at $-L_x < x < 0$, into a free space at $x > 0$ and $x < -L_x$, see Fig. 1. The applied magnetic field, \mathbf{H}_0 , along the z axis is assumed to satisfy $H_{c1} \ll H_0 \ll H_{c2}$, with H_{c1} and H_{c2} being the first and the second critical fields. The transport current along the y -axis results in the motion of the vortex lattice at a speed \mathbf{v} along the x -axis. We consider large \mathbf{v} dominated by the Lorentz force and vortex drag, so that the effect of the surface Meissner current and the back effect of the radiation on the lattice motion can be ignored. To obtain the radiation power we need to find electromagnetic fields, B_z and E_y , at the surfaces $x = 0$ and $x = -L_x$. At $v \ll c$ the fields inside the superconductor can be obtained from a

quasistatic approximation ($\mathbf{r} = x, y, z$):

$$\nabla \times \nabla \times \mathbf{B} + \frac{1}{\lambda^2} \mathbf{B} = \quad (1)$$

$$\frac{\Phi_0 \mathbf{e}_z}{\lambda^2} \sum_{\mathbf{n}} \int dz \delta[\mathbf{r} - \mathbf{r}_n(t, z)] \Theta[-x_n(t)] \Theta[L_x + x_n(t)]$$

$$c[\nabla \times \mathbf{E}] = -\partial \mathbf{B} / \partial t. \quad (2)$$

where \mathbf{e}_z is the unit vector along the z -axis, $\Theta(x)$ is a unit step function, λ is the London penetration length, and $\mathbf{r}_n(t, z)$ is the position of the center of the $\mathbf{n} = (n, p)$ -th vortex line in the vortex lattice. Consider first a square lattice with the lattice spacing $a = \sqrt{\Phi_0/B} \ll \lambda$. We shall assume that the transport current j is sufficient to drive vortices in the laminar (Bragg glass) regime [11, 12]:

$$x_n(t) = an + vt, \quad y_p(z) = ap + \delta_p(z). \quad (3)$$

Here n, p are integers and $\delta_p(z)$ accounts for disorder in the position of vortices along the y -axis. Strictly speaking, vortices do not form an ideal periodic lattice along the x -axis. Rather, in the absence of topological defects, moving vortices adjust to quenched disorder by periodically replacing each other at some favorable points, $x_n(0, p, z)$, with a period $T = a/v$ [12],

$$x_n(t, p, z) = x_n(0, p, z) + [x_{n+1}(0, p, z) - x_n(0, p, z)]f(t).$$

Here $f(t)$ is a periodic function with the period T , satisfying $f(t) \approx t/T$ inside the interval $[0, T]$. One can show, however, that at $a \ll \lambda$ Eq. (3) is sufficient to describe the motion of Bragg glass along the x -axis – the magnetic field at the boundary (that determines radiation) is only weakly affected by disorder in $x_n(0, p, z)$. Phase shifts in otherwise perfectly periodic process are introduced only by time-dependent thermal fluctuations. This is similar to Josephson oscillations, see Ref. 13. The similarity of the vortex-induced radiation to Josephson oscillations also follows from the fact that the washboard frequency satisfies the relation $\hbar\omega = 2eV$, where V is the voltage drop for one period of the vortex lattice in the direction of the current. This is a consequence of the expression for the dc electric field in that direction, $E_y^{(dc)} = (v/c)B$. Note that delta-functions in the right hand side of Eq. (1) correspond to point-like vortex cores. In reality the cores have radius ξ , the superconducting correlation length, which will be accounted for in the following. Here we will ignore thermal fluctuations that lead to the broadening of radiation lines.

For $L_x \gg \lambda$ one can prove that the two surfaces, $x = 0$ and at $x = -L_x$, can be studied independently. We, therefore, begin with considering a semi-infinite superconductor at $x < 0$. The fields inside the superconductor ($x < 0$), must be matched by solutions of Maxwell equations in the free space, ($x > 0$). The total fields inside the superconductor are

$$B_z(t, \mathbf{r}) = B_{zv}(t, \mathbf{r}) + B_{z0}(t, \mathbf{r}),$$

$$E_y(t, \mathbf{r}) = E_{yv}(t, \mathbf{r}) + E_{y0}(t, \mathbf{r}), \quad (4)$$

where B_{zv} and E_{yv} are magnetic and electric fields produced by vortices, while B_{z0} and E_{y0} are solutions of Eqs. (1) and (2) with a zero right-hand side in Eq. (1). For B_{zv} one obtains

$$B_{zv}(\omega, \mathbf{k}) = \sum_{\mathbf{n}} \int dz e^{ik_z z} \frac{\Phi_0}{1 + \lambda_\omega^2 \mathbf{k}^2} \frac{e^{-i\omega a n/v - ik_y y_p(z)}}{i\omega - ik_x v - \epsilon}$$

$$= \sum_{m,p} \int dz e^{ik_z z} \frac{2\pi v \Phi_0}{a(1 + \lambda_\omega^2 \mathbf{k}^2)} \frac{e^{-ik_y y_p(z)} \delta(\omega - m\omega_0)}{i\omega - ik_x v - \epsilon}, \quad (5)$$

where $\mathbf{k} = (k_x, k_y, k_z)$. The summation over n was carried out with the help of the relation

$$\sum_n \exp[-(i\omega a/v)n] = 2\pi \sum_{m=1}^M \delta(\omega a/v - 2\pi m), \quad (6)$$

where the upper limit $M \sim a/\xi$ is due to the nonzero size, ξ , of the vortex core. The frequency dependent London length is given by $\lambda_\omega^{-2} = \lambda^{-2} - k_\omega^2 + 4\pi k_\omega \sigma_q / c$, where σ_q is the quasiparticle conductivity and $k_\omega = \omega/c$. Integrating Eq. (5) over k_x and using inequality $a \ll \lambda$ we obtain the amplitude of the oscillating magnetic field at the boundary $x = 0$ at $\omega = \omega_m = m\omega_0$:

$$B_{zv}(\omega, x=0, k_y, k_z) = \sum_p \int dz \frac{\Phi_0 v e^{i[k_z z - k_y y_p(z)]}}{2i\omega \lambda_{\omega_m} a}. \quad (7)$$

For the electric field of the vortex lattice, with the help of Eq. (2) and $\nabla \cdot \mathbf{E} = 0$ one obtains $E_{yv}(\omega, \mathbf{k}) = (vk_x^2 / (ck^2)) B_{zv}(\omega, \mathbf{k})$. Solutions of the homogeneous equations are $E_{y0}(\omega, x, y) = -ik_\omega \lambda_\omega^2 B_{z0}(\omega, x, y)$ and

$$B_{z0}(\omega, x, y) = \int \frac{dk_y}{2\pi} A(\omega, k_y) e^{(\lambda_\omega^{-2} + k_y^2)^{1/2} x + ik_y y}, \quad (8)$$

where $A(\omega, k_y)$ is determined by the continuity of the tangential components of the fields at the boundaries of the superconductor.

Maxwell equations determine the relation between B_z and E_y everywhere in free space $x > 0$. To find this relation at the boundary, $x \rightarrow +0$, we follow the derivation outlined in Ref. 14. We assume that at $x > 0$ there is *only outgoing* electromagnetic wave. Then the electric field $E_y(\omega, \mathbf{r}) = \int dt \exp(i\omega t) E_y(t, \mathbf{r})$ in free space is completely determined by its value at the boundary through $[\mathbf{k}_\perp = (k_y, k_z)]$

$$E_y(\omega, \mathbf{r}) = \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} E_y(\omega, x=0, \mathbf{k}_\perp) e^{i\mathbf{k}_\perp \cdot \mathbf{r}}$$

$$\times \left\{ \exp[i(k_\omega^2 - \mathbf{k}_\perp^2)^{1/2} \text{sign}(\omega)x] \Theta(k_\omega^2 - \mathbf{k}_\perp^2) \right.$$

$$\left. + \exp[-(\mathbf{k}_\perp^2 - k_\omega^2)^{1/2} x] \Theta(\mathbf{k}_\perp^2 - k_\omega^2) \right\}. \quad (9)$$

The first term in the right hand side is due to radiated waves, while the second term comes from the waves decaying exponentially away from the boundary. Using this relation, Eq. (2)

and $\nabla \cdot \mathbf{E} = 0$, we express the magnetic field in free space via $E_y(\omega, x = 0, y, z)$ and finally obtain the relation between $B_z(\omega, x = 0, y, z)$ and $E_y(\omega, x = 0, y, z)$ at the boundary:

$$B_z(\omega, 0, \mathbf{k}_\perp) = \zeta(\omega, \mathbf{k}_\perp) E_y(\omega, 0, \mathbf{k}_\perp), \quad (10)$$

$$\zeta(\omega, \mathbf{k}_\perp) \equiv \frac{|k_\omega| \Theta(k_\omega^2 - \mathbf{k}_\perp^2)}{\sqrt{k_\omega^2 - \mathbf{k}_\perp^2}} - \frac{ik_\omega \Theta(\mathbf{k}_\perp^2 - k_\omega^2)}{\sqrt{\mathbf{k}_\perp^2 - k_\omega^2}}.$$

For the left boundary, $x = -L_x$, one should reverse the sign of ζ in Eq. (10).

The above relations allow one to determine the fields B_{z0} and E_{y0} and, finally, the total fields B_z and E_y at the boundary as well as the Poynting vector of the radiation. Radiation power at frequency ω outside the superconductor at $x > 0$ (to the right) is given by

$$\mathcal{P}_{\text{rad}}^r(\omega) = \frac{c}{4\pi} \int dydz \operatorname{Re}[E_y e^{-i\omega t}] \operatorname{Re}[B_z e^{-i\omega t}]$$

$$= \frac{c}{8\pi} \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} \operatorname{Re}[\zeta^{-1}(\omega, \mathbf{k}_\perp)] |B_z(\omega, 0, \mathbf{k}_\perp)|^2, \quad (11)$$

where integration is over $|\mathbf{k}_\perp| < |k_\omega|$. Using Eqs. (4) and (10) we obtain for the amplitude of the oscillating magnetic field

$$E_y(\omega, 0, \mathbf{k}_\perp) = \frac{E_{yv}(\omega, 0, \mathbf{k}_\perp) + i\lambda_\omega k_\omega B_{zv}(\omega, 0, \mathbf{k}_\perp)}{1 + i\lambda_\omega k_\omega \zeta(\omega, \mathbf{k}_\perp)} \quad (12)$$

with $\lambda_\omega k_\omega \ll 1$ and $E_{yv}(\omega, 0, \mathbf{k}_\perp) \ll \lambda_\omega k_\omega B_{zv}(\omega, 0, \mathbf{k}_\perp)$ at $\omega = m\omega_0 < \Delta/\hbar$. The power of the radiation into free space to the right or to the left of the superconductor becomes

$$\mathcal{P}_{\text{rad}}^{r,l} \approx \frac{ck_\omega}{8\pi} \int \frac{(d\mathbf{k}_\perp/4\pi^2)}{\sqrt{k_\omega^2 - \mathbf{k}_\perp^2}} \{ |\lambda_\omega k_\omega B_{zv}(\omega, 0, \mathbf{k}_\perp)|^2$$

$$\pm 2\operatorname{Im}[\lambda_\omega k_\omega B_{zv}(\omega, 0, \mathbf{k}_\perp) E_{yv}(\omega, 0, \mathbf{k}_\perp)] \}. \quad (13)$$

We see that the radiation power is slightly stronger to the right (in the direction of the vortex motion), though the difference is small. In the following we neglect this difference.

Substituting the amplitude of the magnetic field at the boundary, $B_{zv}(\omega, 0, \mathbf{k}_\perp)$ of Eq. (7), into the expression (13) for the radiation power, and averaging over disorder in the vortex lattice, we obtain the power,

$$\mathcal{P}_{\text{rad}}(\omega_m) = L_y L_z \frac{\Phi_0^2 v^2 k_\omega}{32\pi c a^2} \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} \frac{S(\mathbf{k}_\perp)}{\sqrt{k_\omega^2 - \mathbf{k}_\perp^2}} \quad (14)$$

$$S(\mathbf{k}_\perp) = \sum_p \int \frac{dz}{a} \exp[ik_z z - ik_y y_p(z)], \quad (15)$$

where $S(\mathbf{k}_\perp)$ is the structural factor of the moving vortex lattice. It is peaked at $k_z = 0$ and $k_y = 2\pi n/a$, and has the following properties: $S(0, 0) = L_y L_z / a^2$, $\int d\mathbf{k}_\perp S(\mathbf{k}_\perp) = (2\pi/a)^2$. If translational correlations in the vortex lattice decay exponentially, then at $L_{y,z}^{-1} \ll k_{y,z} \ll (a\ell_{y,z})^{-1}$ one should use $S(k_y, k_z) = \ell_y \ell_z$, where ℓ_y and ℓ_z are dimensionless transversal and longitudinal correlation lengths expressed in units of the intervortex spacing a ($\ell_z \gg \ell_y$) [15].

Consider first the case of weak disorder, when correlation lengths exceed dimensions of the crystal, $a\ell_y \gg L_y$ and $a\ell_z \gg L_z$, and when the wavelength of the radiation is small compared to the crystal, $c/\omega_m \ll L_y, L_z$. With account of the above properties of the structural factor, integration over \mathbf{k}_\perp in Eq. (14) gives for the radiation power at $\omega = \omega_m$

$$\mathcal{P}_{\text{rad}}(\omega_m) = L_y L_z \frac{v^2 B^2}{32\pi c}, \quad (16)$$

where we neglected insignificant dynamical terms in λ_ω . An amusing feature of the above result is that each electromagnetic mode up to $\omega_m \sim \Delta/\hbar$ satisfying conditions $c/\omega_m \ll L_y, L_z$ makes equal contribution, $v^2 B^2 / (32\pi c)$, to the radiation power per unit square of the radiating surface. The total power, summed up over all modes, is of order $\mathcal{P}_{\text{rad}}^{(\text{tot})} / L_z L_y = (M/32\pi)(v^2 B^2 / c) \sim [1/(16\sqrt{2\pi})](v^2 / c) B^{3/2} H_{c2}^{1/2}$. For weak disorder but small crystal (or lower frequency ω_m), $L_y, L_z \ll c/\omega_m$, we obtain

$$\mathcal{P}_{\text{rad}}(\omega_m) = \left[L_y L_z \frac{v^2 B^2}{32\pi c} \right] \frac{L_y L_z \omega_m^2}{c^2}. \quad (17)$$

Quadratic dependence of the radiation power on L_y and L_z is due to superradiance, that is, coherent radiation by vortices positioned within the radiation wavelength.

Stronger disorder results in smaller correlations lengths, $a\ell_y \ll L_y, (c/\omega_m)$ and $a\ell_z \ll L_z, (c/\omega_m)$. In this case the power of the radiation is reduced:

$$\mathcal{P}_{\text{rad}}(\omega_m) = \left[L_y L_z \frac{v^2 B^2}{32\pi c} \right] \frac{a^2 \ell_y \ell_z \omega_m^2}{c^2}. \quad (18)$$

The coherent radiation takes place within the surface area of size $(\ell_y a)(\ell_z a)$, while the total radiation power is a sum of independent contributions from such areas. Note, that ℓ_y, ℓ_z are correlation lengths for the *moving* vortex lattice. Due to the effect of dynamical reordering [11, 12, 16, 17], they must be greater than the ones for the static lattice. Thus measurements of the electromagnetic radiation from moving vortex lattices provide information on translational correlation lengths and their dependence on velocity.

Static square lattices have been observed in high-temperature superconductors: $\text{La}_{1.83}\text{Sr}_{0.17}\text{CuO}_{4+\delta}$ in fields above 0.4 T [18] and $\text{YBa}_2\text{Cu}_3\text{O}_7$ in fields above 11 T [19]. Square vortex lattices have been also observed in borocarbides [20]. There is little information, however, on whether the rectangular symmetry is preserved in a vortex lattice moving at a high speed. For a triangular vortex lattice x_{np} in Eq. (3) must be replaced by $x_{np} = an + vt + a[1 + (-1)^p]/4$. Then Eq. (5) acquires an additional factor $[1 + \exp(i\pi m)]$ so that only even harmonics, $2m\omega_0$, of the washboard frequency are present in the radiation; their intensity being 1/4 of that for a square lattice. For a vortex liquid the integrals must be dominated by correlations at small distances, that is by k_y of order $1/a$, and the radiation becomes suppressed by a factor $a^2 \omega_m^2 / c^2$, i.e., it is practically absent.

Energy burst from a bundle of $\Lambda = L_x L_y / a^2$ vortices crossing the edge of a superconductor has been discussed by Dolgov and Schopol [10]. They correctly noticed that the radiation spectrum from the vortex lattice consists of discrete lines and can be superradiant. According to Ref. 10, the transient radiation of a single bundle, as in the case of a single vortex, peaks at $(a/\sqrt{3}\lambda)\omega_0$, which is approximately the inverse time needed for the vortex field of diameter λ to cross the crystal edge. We found completely different spectral power of the electromagnetic radiation from a moving vortex lattice.

To obtain significant radiation power one needs high vortex lattice velocity v . This velocity is limited by dissipation (heating). Pulse technique helps to diminish heating and reach higher velocities [18, 19] up to the critical velocity v^* arising from Larkin-Ovchinnikov instability [21]. The latter occurs due to the decrease of the vortex drag coefficient with v as $\eta(v) = \eta(0)(1 + v^2/v^{*2})^{-1}$. Such a behavior of $\eta(v)$ is caused by the escape of quasiparticles from normal cores due to the effect of the electric field. It leads to a negative slope in the I-V characteristics at velocities above v^* . The latter depends on the inelastic scattering of quasiparticles and decreases on cooling. Experimental study [22] of this instability in $\text{Nd}_{1.85}\text{Ce}_{0.15}\text{CuO}_x$ gives $v^* = 8$ m/s at 8 K in the field 120 mT. At such velocity we estimate the radiation power as $2 \mu\text{W}/\text{cm}^2$ at $\omega \sim \Delta/\hbar$ in the field of 1 T for a square vortex lattice in the case of weak disorder, Eq. (16). Much higher value of the critical velocity, $v^* = 1.2 \cdot 10^3$ m/s, was obtained in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ films at 72 K in the field of 1.8 T [23]. At such speed and field the radiation power from a weakly disordered square lattice would be of order $0.1 \text{ W}/\text{cm}^2$.

To obtain the efficiency of the radiation source, we write the dissipation power as $\mathcal{P}_{\text{dis}} = (BL_x L_y / \Phi_0)(\eta v^2 L_z / 2)$, where $B = \Phi_0 / a^2$, $(BL_x L_y / \Phi_0)$ is the number of vortices, and $\eta v^2 / 2$ is dissipation of energy per unit length of the vortex line. For a large superconductor the efficiency, $r = \mathcal{P}_{\text{rad}}^{(\text{tot})} / \mathcal{P}_{\text{dis}}$, in the case of weak disorder, is given by $r = \Phi_0 B / (32\pi c \eta L_x)$. To estimate r one can use the Bardeen-Stephen formula for the drag coefficient, $\eta = H_{c2} \Phi_0 / (\rho_n c^2)$, with ρ_n being the normal state resistivity. At $L_x \sim 10\lambda$ and $B \sim 0.1 H_{c2}$, r can reach 10^{-4} . This means that at a moderate vortex lattice velocity of 20 m/s and a cooling rate of $1 \text{ W}/\text{cm}^2$ the radiation power of $0.1 \text{ mW}/\text{cm}^2$ can be achieved. Note that small r justifies the approach in which the back effect of the radiation on the motion of vortices is ignored.

In Conclusion, coherent electromagnetic radiation should accompany the flux-flow state of a moving vortex lattice. The spectrum of the radiation has discrete character and extends up to the frequency corresponding to the superconducting gap. Thus, in principle, this effect can be used to generate radiation in the terahertz frequency range. Radiation power depends strongly on the vortex lattice symmetry, velocity and degree of translational order, providing a possible tool for character-

ization of moving lattices.

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